Considering "Golden Angles" as a Geometric Principle in the Paintings and Drawings of Leonardo da Vinci

Lin Kuang[1]

As early as the fifth century before the Common Era, the Greek philosopher and mathematician Pythagoras revealed how the ideal proportions of the human body conform to principles of a so-called "golden ratio" where, in the epigrammatic words of mathematician Euclid of Alexandria around 300 B.C., "the whole line is to the greater segment as the greater is to the lesser."

While none of Pythagoras writings are known still to exist, his near contemporary, the architect and sculptor Phideas, is believed to have followed Pythagoras' golden ratio principles in his friezes and statues for the Parthenon in fifth century B.C.; and the sculptor Polykleitos the Elder (c. 450-420 B.C.) also described, in his eminent *Canon*, a mathematical approach to rendering the ideal human form in ratios and proportions that scholars believe embodied this influence of Pythagoras.[2]

In the modern era, many scholars have postulated that, during the Renaissance, Leonardo da Vinci (1452-1519) employed these golden ratio and other mathematical and geometric principles in his paintings, drawings and other works. It is often noted, for example, that his works appear to confirm "golden rectangles" in their proportion and dimension. This paper discusses a related but distinct geometric phenomenon that may also be found in paintings and drawings of Leonardo da Vinci: "golden angles."

Leonardo and the Golden Ratio

It is well known that, following his apprenticeship in the studio of painter Andrea del Verrocchio, Leonardo studied under the seminal mathematician Luca Pacioli in the 1490s, copying the master's proportional and mathematical tables, and that he illustrated Pacioli's 1509 treatise *De Divina Proportione*, including drawings of a number of complex geometrical polyhedra.

In that treatise, Pacioli called the golden ratio a "divine" mathematical proportion that virtually defines beauty in the human face. Pacioli also discussed there the use of linear perspective by Leonardo's contemporary, the painter Piero della Francesco (1415-1492), who around 1480 had authored *De Prospectiva Pingendi (On Perspective for Painting)*, a work grounded in mathematics and geometry.[3] Clearly, Leonardo was familiar with theories of both golden ratio and linear perspective.

"Let no one who is not a mathematician read my principles," Leonardo famously wrote in his *Notebooks*, describing art as "a science." Perhaps best-known of this investigative approach is *Vitruvian Man*, or *Le proporzioni del corpo umano secondo Vitruvio* (c. 1490), a pen-and-ink

[1] Ideas expressed herein are the author's own. The author wishes to thank Jennifer L. Hall, M.I.S., for invaluable suggestions, writing, research and editing.

[2] *See, e.g.*, J.E. Raven, *Polyclitus and Pythagoreanism*, CQ (1951) 147-52.

[3] *See, e.g.*, Bulent Atalay, *Math and the Mona Lisa: The Art and Science of Leonardo da Vinci* 86 (Washington, D.C.: Smithsonian Books, 2004).

drawing that is Leonardo's study of geometric proportions of the human body as set forth in the treatise *De Architectura* by ancient Roman architect Vitruvius in the late first century B.C. Vitruvius believed these human proportions formed the very basis of classical architecture. It was by employing these symmetrical proportions, Vitruvius wrote, "that the famous painters and sculptors of antiquity attained to great and endless renown."[4]

In Renaissance Italy, artists and intellectuals rediscovered classical Greek and Roman art and literature, and the "desire to know and to match the excellence of the ancients often engendered passionate endeavor."[5] Leonardo was well familiar with aesthetic teachings of Plato, with their emphasis on proportion and harmony, and himself emphasized the direct observation of nature in all its variation that would epitomize the later Renaissance.[6] When Raphael painted his magnificent fresco *The School of Athens* (c. 1509-11) for Pope Julius II (Vatican, Stanza della Segnatura), he is believed to have based his portrait of Plato, pointing to heaven and walking beside Aristotle, upon a likeness of his own contemporary Leonardo, and included portraits of Pythagoras, pen and notebook in hand, and Euclid with compass, on the marble steps.

While not universally accepted,[7] it is often suggested that in addition to other geometric proportions and perspectives, Leonardo employed golden ratio principles in his own timeless masterpieces. For example, it has been observed that proportions of a "golden rectangle" can be superimposed from head and upper chest to bodice of his portrait subjects, in both *Ginevra de Benci* (c.1474) (*see* Fig. 7) and *The Lady with Ermine* (c.1489-90) (Fig. 9). It has been suggested that the face of the *Mona Lisa* (c. 1503-05) (Fig. 8) forms a golden rectangle, and her figure is generally framed by a "golden triangle." Figures in *The Virgin of the Rocks* (c.1483-86) (Fig. 6) are thought to form a "golden pyramid." A number of golden rectangles are discernible in the composition of *The Last Supper* (c.1494-98) (Fig. 1), a work that is acknowledged to have demonstrated mastery of linear perspective. That he may have sought to integrate varied geometric principles within each single work, and would be motivated to do so, would seem compatible with all we know about Leonardo da Vinci.

Fig. 1 *The Last Supper*, Santa Maria delle Grazie, Milan).

[4] Vitruvius, *On Architecture* 3.1.
[5] Department of European Paintings, The Metropolitan Museum of Art, *The Rediscovery of Classical Antiquity* (New York: 2002).
[6] *See* Istituto Geografico de Agostini, *Leonardo da Vinci* 36 (1939, republished 1956); Chad Trainer, "Finding a Philosophy in Leonardo," *Philosophy Now* (2005).
[7] *See, e.g.*, Mario Livio, *The Golden Ratio* 162-166 (New York: Broadway Books, 2002).

Standard "Golden Ratio" Analysis

To understand "golden angles", it is first necessary to understand golden ratio. Under standard golden ratio analysis, the parts of a whole are held to be in perfect, golden proportion to each other when the whole (a + b) divided by a larger segment (a) equals the larger segment (a) divided by the smaller segment (b). Where this perfect proportion is present, then value (a) divided by value (b) will always equal the number 1.618, which is the golden ratio. The converse is equally true: where value (b) divided by value (a) equals the number 0.618, then those two values are held to be in perfect proportion to each other.

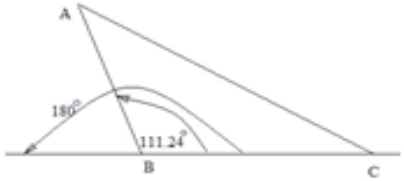
$$\frac{a+b}{a} = \frac{a}{b}. \quad \textit{or} \quad \frac{a}{b} = \frac{b}{a-b}.$$

It is traditionally noted that these pleasing proportions are used in architecture and art in the form of golden rectangles, golden triangles, and other geometric shapes. For example, a "golden rectangle" is a rectangle whose side lengths are in golden ratio to each other; in other words, the longer length divided by the width is equal to 1.618, and the length equals the width multiplied by 1.618. A square inserted into a golden rectangle will produce a smaller golden rectangle in the same proportion (i.e., whose side lengths are in golden ratio in relation to each other) and so on; these proportions are discernible in the architectural design and décor of the Parthenon and, many believe, in the *Mona Lisa*. Similarly, a "golden triangle" is an isosceles triangle in which two identical sides are in golden ratio to the remaining side (the base). Since the sum of the three angles of any triangle is always 180 degrees, where a congruent side divided by the base equals the number 1.618, the two base angles of the triangle will be 72° angles, and the vertex angle will always be a 36° angle. This is the traditional "golden triangle."

Related Phenomena: "Golden Angles" and "Golden-angled Triangles"

A related but distinct geometric phenomenon, that also incorporates the golden ratio, may be found in paintings and drawings of Leonardo da Vinci. That is, a significant number of important compositional angles and diagonals are formed at 111.24° relative to a 180° angle (*see* Fig. 2). In geometry, these compositional angles are known as "golden angles", based on the fact that a 180° angle (which forms a straight line, or the sum of the three angles of any triangle) divided by that particular angle of 111.24° equals the number 1.618, or the golden ratio.

Fig. 2 A golden angle: 180°/111.24°= 1.618, the golden ratio.



This "golden angle" of 111.24° creates a stable and satisfactory element within a composition, and is known and employed today to produce reliable and efficient results in science and engineering. Moreover, a third segment that resolves or "subtends" any two segments meeting at a 111.24° angle will naturally complete a triangle that may be called a "golden-angled triangle," based again on the fact that 180° divided by 111.24° equals 1.618. In contrast to the traditional concept of the "golden triangle" (whose side *segments* are in golden ratio to each other and whose two base angles will each be 72°), a "golden-angled triangle" will be defined to have one angle of 111.24° and will be formed by a third segment that subtends that 111.24° angle; the degree of the other two angles will vary based on the length of the segments.

Rather than a ratio of relative *segments* as in traditional golden ratio analysis, golden angles and golden-angled triangles imply a ratio of *angles*. While segments (or lines) are defined by two points and may be expressive of static proportion, harmony and beauty, the golden angle 111.24°, like all angles, is defined by three points and is therefore expressive of dynamic beauty. It is also expressive of three-dimensional directional or "azimuthal" space, relative to the common navigational 180°, 360° and 90° ranges.

It is possible, therefore, to have a golden ratio of angles other than 180°/111.24°, so long as angle A divided by angle B equals 1.618. For example, in *Lady with an Ermine* (*see* Fig. 9), a special 55.6° angle is arguably formed by the turn of the lady's head relative to her torso. To the extent that a full front-facing pose creates a 90° angle and a three-quarter profile pose is traditionally considered to create a 45° angle, the pose in *Lady with an Ermine* may be deemed to create a ratio of angles that equals the golden ratio (90° /55.6°= 1.618). Leonardo carefully echoed this elegant turn of the lady's head by her shoulder and by the profile of the ermine in her arms.

In itself, the concept of a "golden angle" that incorporates the golden ratio is not new. Golden angles have been recognized in geometry and in nature, and applied in the engineering sciences, in some cases for centuries.

In geometry, for example, it is understood that a circle may be divided in proportion to the golden ratio, in that its circumference of 360° (which is twice 180°) divided by a "pie slice" angle of 222.49° (which is twice the golden angle of 111.24°) equals the golden ratio 1.618. The remaining smaller "pie slice" angle of 137.51° is considered to be a "golden angle" relative to the larger angle of 222.49° (in that 222.49 divided by 137.51 equals 1.618), which in turn is in golden ratio relative to the whole circumference of 360°, thereby satisfying Euclid's dictum. Another way of stating this is that the golden angle of 111.24° divides a semi-circle according to the golden ratio.

In nature, it was observed in the 1800s that each new leaf on the stem of certain plants will position at angles of about 137.51° relative to the previous one, such that the angle between a line from stem to leaf and a corresponding line to the next leaf will generally be 137.51°. The resulting numbers of spirals or petals will reflect the sequence of numbers associated with mathematician Leonardo Pisano ("Fibonacci") (c. 1170–1240) that reflect the golden ratio, in that each number is equal to the sum of its preceding two numbers and the ratio of any two such

numbers in the Fibonacci sequence is in the golden ratio.[8]

More recently, in engineering, a number of sophisticated applications have been found for the golden angle in imaging and modeling technologies, such as magnetic resonance imaging (MRI) and computed tomography (CT) scanning, as well as 3-D animation and modeling software for the graphic arts. For example, the Institute of Electrical and Electronics Engineers (IEEE) has published research on the use of "constant azimuthal radial profile spacing of 111.24°" in so-called golden angle radial magnetic resonance imaging, and on the use of smaller irrational angles based on the Fibonacci sequence called "smaller golden angle surrogates" to be applied for reliable and dynamic imaging of the heart.[9]

Just as engineers today use profile spacing of 111.24° in golden angle imaging, so Leonardo as mathematician and engineer naturally may have manifested such angles in his drawings and paintings to create three-dimensional directional depth and dynamic movement. An azimuth, from the Arabic mathematicians and navigators studied by Fibonacci, is a measure of angles in a spherical coordinate system with which Leonardo, with his interest in astronomy, navigation, and mapmaking, would be familiar. Not only did Leonardo create the first illustrations of complex polyhedra for *De Divina Proportione*, he also produced maps in the early 1500s of unprecedented accuracy, detail and perspective.

A great number of Leonardo's paintings, such as *Annunciation* (c.1472-75), *The Virgin of the Rocks* (c.1483-86), *The Lady with an Ermine* (c.1489-90), *Ginevra de Benci* (c.1474), and others, seem to employ these "golden angles" and "golden-angled triangles" to form significant elements of their composition. Points of obvious visual importance in the composition fall upon and form the vertices of these angles, and of the triangles that resolve them, thereby defining the relative positions of those points of significance. These angles embodying the golden ratio contribute to achieving an inherent sense of stable visual harmony and three-dimensional verisimilitude.

The compositional angles sometimes contain minor fluctuations on or around 111.24 degrees. Such deviations are reasonable, as the point is not that Leonardo utilized precise measurements to create profound humanist works of art. Rather, on an intuitive and perhaps

---

[8] *See* Md. Akhtaruzzaman and Amir A. Shafie, "Geometrical Substantiation of *Phi*, the Golden Ratio and the Baroque of Nature, Architecture, Design and Engineering," *International Journal of Arts* 1(1): 1-22 (2011). Scientist Wilhelm Hofmeister reported in 1868 that, for efficient use of sun and rain, plants forming in spirals, such as pinecones, sunflowers and certain cacti and succulents, will form successive primordia along their growing meristem radially outward at a rate proportional to the stem's growth, usually resulting in an angle that converges to a constant value of 137.5 degrees. *See* ibid. *citing also* S. King, F. Beck and U. Luttge, "On the mystery of the golden angle in phyllotaxis," *Plant, Cell and Environment* (Blackwell Publishing Ltd.: 2004) 27, 685-695.

[9] S. Wundrak, J. Paul, J. Ulrici, E. Hell, V. Rasche, "A Small Surrogate for the Golden Angle in Time-Resolved Radial MRI Based on Generalized Fibonacci Sequences," *IEEE Transactions on Medical Imaging*, Vol. X, No. X (2014). *See also* S.F. Liew, Heeso Noh, J. Trevino, L. Dal Negro, Hui Cao, "Localized photonic band edge modes and orbital angular momenta of light in a golden-angle spiral," 2012 Conference on Lasers and Electro-Optics (2012); Wen Hou, Cishen Zhang, "Analysis of compressed sensing based CT reconstruction with low radiation," 2014 International Symposium on Intelligent Signal Processing and Communication Systems (2014) (showing that low radiation computed tomography (CT) scan reconstruction can be achieved with random detectors and golden-angle scanning); Yao Xie, "Comparison of Golden Ratio Based and Random Radial Profile Order for Projection MRI," Reports for EE 391 (Spring 2007 – 2008) (the golden ratio based profile order strategy facilitates use of radial MRI in dynamic study, allowing faster reconstruction and tracking of a moving object.").

conscious level, works of this great Renaissance master consistently manifested a number of "golden angles" to resolve problems of proportion and perspective, and to infuse the works with dynamic and architecturally consistent gesture and movement, through an innate three-dimensional space and harmonic composition.

Significant compositional golden angles manifested in Leonardo's paintings are too numerous for complete analysis in one paper. This paper focuses on a few select, acknowledged masterpieces.

Vitruvian Man

As noted, *Vitruvian Man* (c. 1490) was Leonardo's pen-and-ink study of the geometric proportions of the human body set forth by ancient Roman architect Vitruvius. In his treatise on architecture, Vitruvius addressed "symmetrical relations" of parts "to the general magnitude of the whole."

Vitruvius believed that "the navel is naturally the exact center of the body." He believed that, from a circle centered at the navel of a man, with arms and legs extended, "his fingers and toes will be touched by the circumference." Also a "square will be found described within the figure," he wrote, "if we measure from the sole of the foot to the top of the head" and "apply the measure to the outstretched hands" in that "the breadth will be found equal to the height."[10]

As illustrated in Leonardo's famous drawing, the circle and square are not centered on the same point. Leonardo realized that, with the circle centered at the navel, the arms outstretched horizontally (as for the square figure) would not "touch the circumference of the circle" as Vitruvius had described. Thus, Leonardo illustrated Vitruvius's description of the circle *by drawing the arms raised* to a position where the fingertips align with the the top of the head, rather than at the lower angle Vitruvius had described for the square figure, where outstretched arms form horizontal lines. Leonardo wrote in his accompanying text: "if you open your legs enough that your head is lowered by one-fourteenth of your height and raise your hands enough that your extended fingers touch the line of the top of your head, know that the centre of the extended limbs will be the navel, and the space between the legs will be an equilateral triangle".

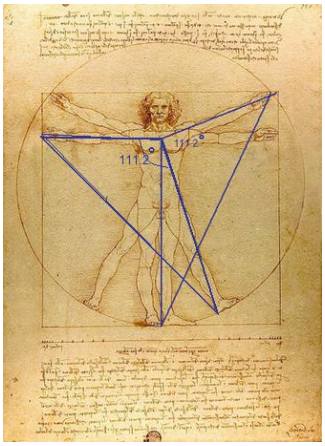


Fig. 3 *Vitruvian Man* (Gallerie dell' Accademia, Venice).

Leonardo's rendering, showing two superimposed figures within the circle and the square, created four solid, implicit "golden-angled triangles" each anchored on "golden angles" on or about 111.24°. The first pair of golden-angled triangles is formed along the strong vertical line of Vitruvian man out to each raised, extended arm (the golden angle), and the solid implied resolving line of that triangle. The second pair of golden-angled triangles is formed along the strong horizontal of his outstretched arms out to the line of each leg opened "enough that your

[10] Vitruvius, *On Architecture, Book III, Ch. I - The Planning of Temples*, translated by F. Granger 161 (Boston: Harvard University Press, 1983).

head is lowered by one-fourteenth of your height" (the golden angle) and the implied resolving line of that triangle. (*See* Fig. 3, showing two of these four golden angles.) These deeply intuited golden angles are among the elements of this brilliant drawing that lend it the powerful and timeless pentagrammatic quality to which generations have responded.

Annunciation

The *Annunciation* is a work of particular interest as it may reveal how Leonardo, early in his career, experimented with golden ratios including golden angles in the pursuit of unifying art and science. A study of the *Annunciation* may help lay groundwork for understanding the presence and purpose of golden angles in his later works.

The *Annunciation* held at the Uffizi Gallery in Florence is considered Leonardo's first complete work after leaving the shop of his master Andrea del Verrocchio. The painting has occasionally been noted for some compositional flaws attributed to Leonardo's relative youth at the time of its painting. In fact, some of these asserted flaws could be explained by Leonardo's attempt to apply golden ratio principles and to synthesize them with principles of perspective. Thus, this painting could more properly be seen as an early experiment by Leonardo toward unifying art and science, that led to some of its seeming deficiencies.

In *Annunciation*, four golden-angled triangles ground the composition (Fig. 4, in blue, numbered 1-4), along with other shapes that also meaningfully apply the golden ratio. The first golden angle prominently defines the overall compositional relationship, connecting the Virgin Mary, the Angel Gabriel, and the base of the lecturn on which Mary rests her book. A second golden-angled triangle connects these three main compositional elements, acting to create an illusion of depth and three-dimensional space. The third and fourth golden angles respectively create the fundamental forms of the Virgin Mary and of the Angel Gabriel (who themselves together suggest a solid compositional base of a triangle whose top vertex is out of the picture frame. The Virgin Mary forms a traditional "golden triangle" of her own with base vertices of 72° (*see* Fig. 4, in red).

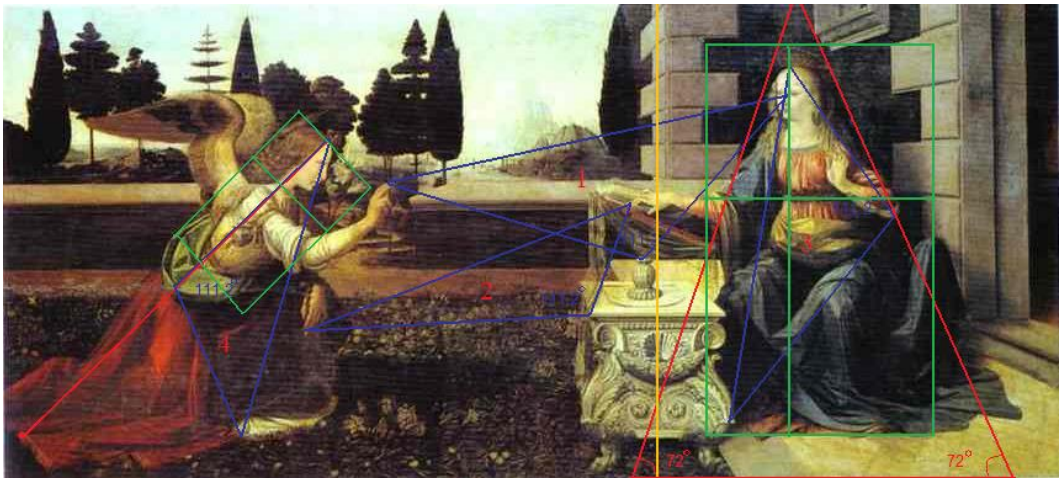


Fig. 4 *Annunciation* (Uffizi Gallery, Florence). Golden-angled triangles in blue, 1-4. Golden ratio width, yellow vertical line.

Annunciation has been criticized as "not entirely successful in its handling of perspective and anatomy," with Mary "seated too far back for her right arm to reach the left side of the book on her reading stand."[11] Yet Leonardo, perhaps realizing implicit power of the golden ratio, employed a golden-angled triangle to connect Mary, Gabriel, and the book lecturn, an important symbol of Christian faith, also allowing the ornate sepulchral table base to be placed directly at the golden ratio width of the *Annunciation*'s golden rectangle (*see* Fig. 4, yellow vertical line). Thus, Leonardo's misjudgment of "perspective and anatomy" may have been the price he paid for his ambitious experiment to harmonize the most visually and symbolically important points of *Annunciation* by applying golden ratios, including golden angles. Application of golden ratio principles may help explain why the painting's unusual composition is held so satisfying despite apparent imperfections.

This explanation becomes more plausible when one compares the *Annunciation* at Uffizi with the version at the Louvre (Fig. 5). The Louvre version, attributed at times to Leonardo and at times to Lorenzo di Credi, also includes the placing of a large stand between Mary and Gabriel. Yet this stand is not connected to Mary and Gabriel by golden angles, and does not appear to carry symbolic importance. Rather, it appears to have been included without clear understanding of why it should be there, resulting in a large inanimate object that serves merely to separate Mary and Gabriel. Golden-angled triangles can be roughly identified on both virgin and angel, but even then, their bottommost vertices do not appear to fall on points of obvious significance (*see* Fig 5). This suggests that, whatever golden ratio applications still can be found in di Credi's version, they were neither deliberate nor intuited, and the Louvre version likely is the more derivative work.

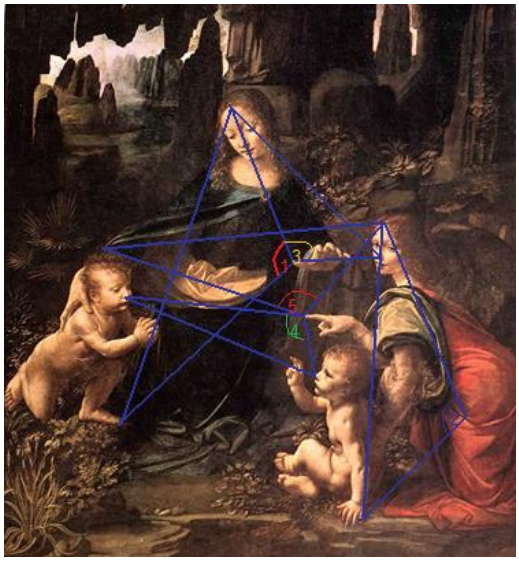
Fig. 5 Annunciation (Musée du Louvre, Paris)

<u>Golden-angled triangles in Leonardo's Other Masterpieces</u>

[11] *See* Pietro C. Marani, *Leonardo Da Vinci*: *The Complete Paintings* 48 (New York: Harry Abrams, 2003) ("In the Uffizi Annunciation, Leonardo tried for the first time to use everything that he had learned in Verrocchio's shop in a single large work. The painting is not entirely successful in its handling of perspective and anatomy. Close examination and x-ray reveal an enormous number of pentimenti, or revisions in the underdrawing and in the incising and priming of the lines that define the architectural forms ... an indication of the artist's still imperfect ability to organize a scene around two figures placed in a carefully defined space. The angel's head was originally titled farther toward the ground, with wings placed higher and on diagonal, as if the figure were moving from the background toward the foreground. The Virgin's placement in space is also flawed; she is seated too far back for her right arm to reach the left side of the book on her reading stand; the lectern itself is very close to the picture plane, and therefore at quite a distance from her.").

In Leonardo's mature works, points of obvious visual significance are linked by golden angles and golden-angled triangles to help establish profound visual harmony. As Leonardo applied principles of linear perspective, he appears also to have mastered the projection of three-dimensional golden angles existing in a virtual three-dimensional space on a painting's two-dimensional surface plane.

This mature application of golden angles is illustrated in *The Virgin in the Rocks* (Fig 6a and 6b). In the version at the Louvre in Paris (Fig. 6a), for example, golden-angled triangles are comprised of focal points located at different levels of three-dimensional "depth." By comparison, the London version, long considered to have been painted in great part by Leonardo's assistants and whose attribution to Leonardo remains in some controversy, shows lesser accuracy in the application of golden angles (and fewer of them, due in part to the absent pointing finger near the center, which forms a key symbolic vertex of a golden angle, as with the angel Gabriel's gesture in the *Annunciation*) resulting in a less harmonious and dynamic compositional structure overall.



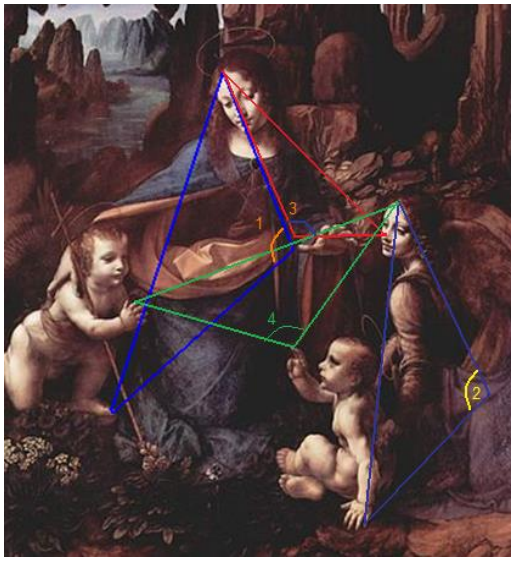
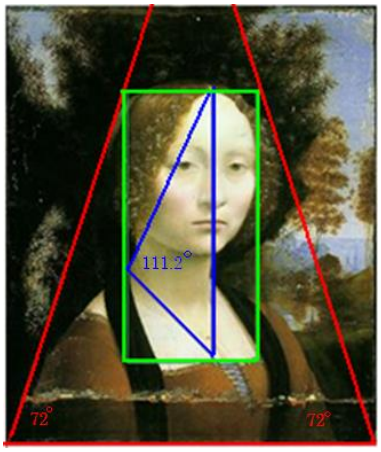


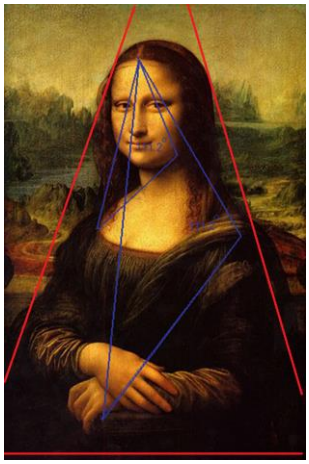
Fig. 6a (Left). *The Virgin of the Rocks* (Musée du Louvre, Paris).
Fig. 6b (Right). *The Virgin of the Rocks* (National Gallery, London).

Turning to other celebrated works, as noted it has been observed that proportions of a "golden rectangle" can be superimposed from the head and upper chest to bodice of both the *Ginevra de Benci* (c.1474) (Fig. 7) and *The Lady with an Ermine* (c.1489-90) (Fig. 9). The face of the *Mona Lisa* (c. 1503-05) (Fig. 8) is believed to form a golden rectangle, and her figure is generally framed by a "golden triangle." In addition, in the *Ginevra*, a traditional golden triangle frames the subject, and a *golden-angled triangle* cuts into the three-dimensional space from her head to bodice to the collar at the nape of her neck, defining the overall three-quarter profile view in a geometrically consistent three-dimensional manner. The *Mona Lisa* displays similar golden angles defining three-dimensional space.

In *Lady with an Ermine*, to the extent that a full-front pose creates a 90° angle, and a three-quarter profile pose is traditionally considered to create a 45° angle, the lady's pose creates a ratio of angles that equals the golden ratio (90° /55.6°= 1.618). (*See* Fig. 9). This elegant turn

of the lady's head is echoed not only by her shoulder, but by the head profile of the ermine, suggesting that, however intuitive, there was nothing arbitrary about the compositional choice.





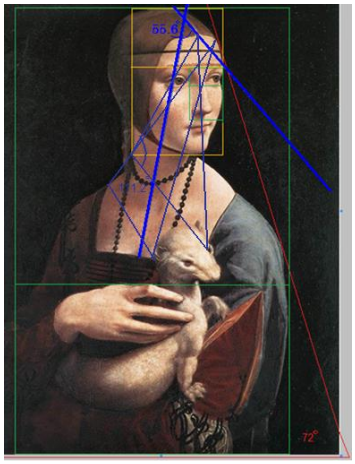
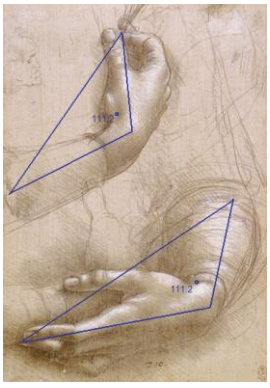


Fig. 7 (Left) *Ginevra de' Benci* (before restoration). (National Gallery of Art, Washington, D.C.).
Fig. 8 (Middle) *Mona Lisa.* (Louvre, Paris).
Fig. 9 (Right) *Lady with an Ermine* (Czartoryski Museum, Kraków).

It has long been speculated that *Ginevra de' Benci*, like the *Mona Lisa*, once had a pair of hands folded in her lap, for which a study drawing is believed to exist at Windsor Castle.[12] At the time of acquisition of the *Ginevra* by the National Gallery of Art in 1967, the Gallery's then-director John Walker understood that the poplar panel had been cut along its bottom edge some time prior to 1780, damage that a cleaning in 1990 revealed more clearly and that was then restored.[13] Digital images have been produced extrapolating what *Ginevra* would look like if it contained a pair of hands; at least one such effort has sought to do so using the golden section. However, Leonardo painted both the *Ginevra* and *Mona Lisa* within the proportions of a golden triangle (*see* Figs. 7 and 8). Given that *Ginevra* already sits within a golden triangle and the cut to the poplar panel is several inches above its bottom edge, it is not necessary to conclude that the figure once had folded hands like the *Mona Lisa*.



[12] Eric Gibson, "Leonardo's Ginevra de' Benci: The restoration of a renaissance masterpiece" 161-62 *Apollo* (London: March 1991), *citing* Kenneth Clark and Carlo Pedretti, *The Drawings of Leonardo da Vinci*, Vol. 1 no. 12,558 (1968).
[13] Ibid. 161-62 *Apollo* (London: March 1991). *See also* John Walker, `*Ginvera de' Benci by Leonardo da Vinci*', National Gallery of Art, Studies in the History of Art, 1967 (Washington, D.C.: 1968).

Fig. 10 (Left) Golden-angled triangles in *Benois Madonna* (Hermitage Museum, St. Petersburg).

Fig. 11 (Middle) Golden-angled triangles in *The Study*

Fig. 12 (Left) Golden-angled triangles in *Adoration of the Magi* (Uffizi, Florence).

Further examples of apparent golden angles, golden-angled triangles, and other apparent application of golden ratios in Leonardo's works, serving similar functions and purpose, may be seen in the *Benois Madonna*, *The Study of Hands*, and *Adoration of the Magi*.

Conclusion

What conclusions may be drawn from observation of a prevalence of golden angles and golden-angled triangles in the paintings and drawings of Leonardo da Vinci? Identification of such properties might assist in authentication and attribution of paintings by the Renaissance genius. The making of a masterpiece is not derived from any one particular element. The presence of 111.24° golden angles is not sufficient condition to determine the authenticity of a painting as one by Leonardo da Vinci, but it may be a probable or even necessary one. It has been observed that, while Leonardo left abundant material in both his drawings and notes on ratios and proportions, it is "impossible, based on them, to establish a Leonardesque rule on proportions."[14]

Fig. 13. Golden angle in *Head of Leda.*

Rather, in the final analysis, Leonardo created timeless works of art from both scientific understanding and a deep humanism, and a keen natural sense of harmony and proportion. Leonardo knew instinctively that art required the application of certain principles and rules, and then the departure from them: For "even if you should wish to make your figures according to one and the same measure, know that one will not be distinguishable from the other, and this is not seen in nature," he wrote.[15] The rules were simply a "corroboration of created form" and therefore of God. From the "scientific and true principles of painting," he wrote in his *Treatise on Painting*, "is then born the actual creation, which is far superior in esteem to the contemplation or science which precedes it."[16] For Leonardo, this truly was the "divine proportion."

[14] Istituto Geografico de Agostini, *Leonardo da Vinci* 447 (New York: Reynal & Company, Inc., 1939).
[15] *Quoted in* Istituto Geografico de Agostini, *Leonardo da Vinci* 447 (New York: Reynal & Company, Inc., 1939, republished 1956).
[16] *Quoted in* ibid., 33-34.